Time Reversal Space-Time Block Coding vs. Transmit Delay Diversity — A comparison based on a GSM-like system

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Abstract - In [1], a new transmit diversity scheme for channels with intersymbol interference was introduced. The scheme is a generalization of the space-time block coding introduced by Alamouti in [3] to channels with intersymbol interference.

The scheme uses two transmit and one receive antenna and achieves the same diversity benefit at the user terminal as can be achieved by using one transmit and two receive antennas. The scheme thus achieves the full available diversity benefit.

In this paper we evaluate the performance of this scheme by applying it to a GSM-like system and comparing it with a delay diversity scheme.

The method proposed in [1] has better performance than transmit delay diversity. Further, as the memory in the overall channel is not increased, the complexity of the equalizer is not increased. This is especially valuable if the receiver uses a maximum likelihood sequence estimator (MSLE). The complexity of the receiver will then be significantly reduced with respect to the delay diversity scheme.

I INTRODUCTION

A way to improve the quality and the data rates in wireless communication is to use multiple antennas at the receiver in order to benefit from diversity. The diversity obtained from multiple antennas helps to combat the fading channel. However, using multiple antennas at the user terminal has the drawback that it makes the subscriber unit larger and more expensive. We are therefore interested in using transmit diversity techniques where multiple antennas are employed at the transmitter instead of at the receiver.

In [3], Alamouti presented a transmit diversity scheme that by utilizing *two transmit* and only *one receive antenna* achieves the same diversity benefit at the user terminal that would be achieved by using *one transmit* and *two receive antennas*. Alamouti scheme is however primarily designed for channels without intersymbol interference. In [1], a generalization of this algorithm to channels with intersymbol interference is presented.

In this paper we evaluate the performance of this scheme by applying it to a GSM-like system and comparing it with the performance of a transmit delay diversity scheme [4].

The paper is organized as follows. Section II introduces the specific notations used in the paper. Section III briefly reviews the time reversal space-time block coding scheme (TR-STBC) introduced in [1], while Section IV describes the transmit transmit delay diversity scheme. In Section V the two methods are compared in term of performance and complexity. Finally, in Section VI we compared the performance of the two transmit diversity methods in terms of bit-error rate (BER) as a function of the signal to noise ratio (SNR), for a GSM-like system.

II NOTATION

Throughout the report, we will consider discrete-time channel models and detectors. A discrete-time filter will be represented as a polynomial in the unit delay operator, q^{-1}, as exemplified below:

$$\begin{aligned} v(t) &= a(q^{-1})u(t) = \left(a_0 + a_1 q^{-1} + \cdots + a_{na} q^{-na}\right) u(t) \\ &= a_0 u(t) + a_1 u(t-1) + \cdots + a_{na} u(t-na), \end{aligned}$$

where na is the *order* of the polynomial for a filter with $na+1$ taps. The discrete time is denoted with the discrete variable t. Note that filters may also be non-causal and have terms with powers of the unit advance operator q.

Multiple-input-single-output (MISO) filters will be represented as polynomial row vectors, and single-input-multiple-output (SIMO) filters will be represented as polynomial column vectors. Multiple-input-multiple-output (MIMO) filters will be represented as polynomial matrices.

The *complex conjugate-time reverse* of a filter $a(q^{-1})$ is defined as

$$\left(a(q^{-1})\right)^{\dagger} \triangleq a^*(q) = a_0^* + a_1^* q + \cdots + a_{na}^* q^{na}. \quad (1)$$

while the *complex conjugate* of a filter $a(q^{-1})$ is defined as

$$\left(a(q^{-1})\right)^{*} = a^*(q^{-1}) \triangleq a_0^* + a_1^* q^{-1} + \cdots + a_{na}^* q^{-na}. \quad (2)$$

Note that the following relation holds:

$$\left(a^*(q^{-1})\right)^{\dagger} = a(q) = a_0 + a_1 q + \cdots + a_{na} q^{na}. \quad (3)$$

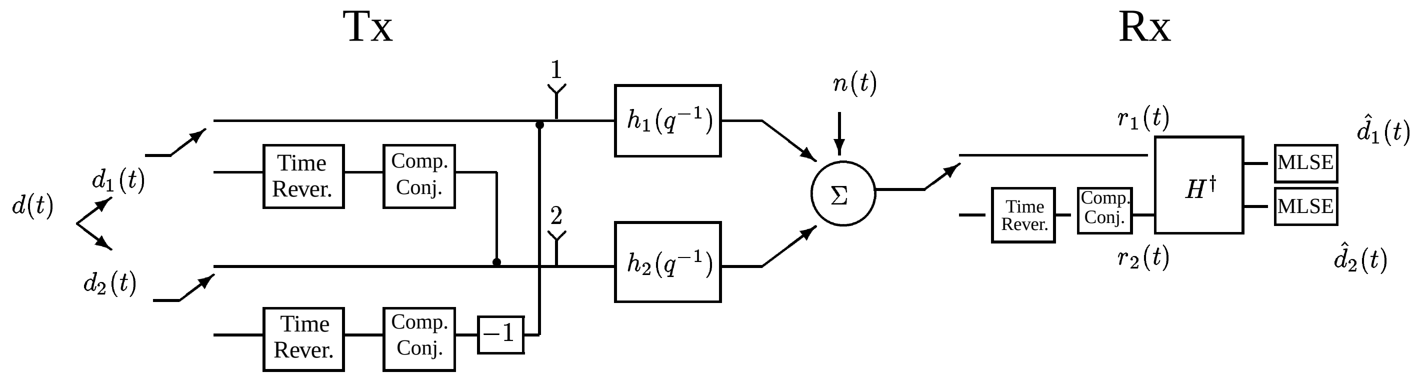

Figure 1: *Schematic view of the transmission in the TR-STBC scheme*

Further, when applied to a polynomial matrix, the operator $(\cdot)^\dagger$ implies transposing followed by complex conjugation and time reversal as in (1).

III TIME REVERSAL SPACE-TIME BLOCK CODING (TR-STBC)

In this section we briefly review the time reversal space-time block coding scheme (TR-STBC) as introduced in [1]. In particular we recall the channel modeling and the special signaling scheme of this method. Figure 1 shows the block diagram of this transmit diversity technique.

A *Channel Modeling*

Let us split the transmitted symbol sequence, $d(t)$, and the received one, $r(t)$, in two halves and let us define the vectors

$$\boldsymbol{r}(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix}, \quad \boldsymbol{d}(t) = \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix}. \tag{4}$$

Let us assume that we transmit in such a way that the received signal $\boldsymbol{r}(t)$ can be expressed as[1]

$$\boldsymbol{r}(t) = \boldsymbol{H}(q, q^{-1})\boldsymbol{d}(t) + \boldsymbol{n}(t), \tag{5}$$

where

$$\boldsymbol{H}(q, q^{-1}) = \begin{bmatrix} h_1(q^{-1}) & h_2(q^{-1}) \\ h_2^*(q) & -h_1^*(q) \end{bmatrix}, \tag{6}$$

and where $\boldsymbol{n}(t)$ is the noise vector, assumed here to be spatially and temporally white with spatial-temporal autocovariance matrix $\boldsymbol{R_{nn}}(q, q^{-1}) = \sigma_n^2 \boldsymbol{I}$. Note that the channels $h_2^*(q)$ and $h_1^*(q)$ have complex conjugated coefficients and are time reversed and thus anti-causal. In the next section we will see how this signaling can be achieved.

The polynomial channel matrix $\boldsymbol{H}(q, q^{-1})$ is orthogonal in the sense that

$$\boldsymbol{H}^\dagger(q, q^{-1})\boldsymbol{H}(q, q^{-1}) = \left(h_1^*(q)h_1(q^{-1}) + h_2^*(q)h_2(q^{-1})\right)\boldsymbol{I}.$$

[1]Note that the pulse shape used in the modulation is part of the overall channel modeled in (6).

In the receiver we filter the received signal $\boldsymbol{r}(t)$ with the matched filter $\boldsymbol{H}^\dagger(q, q^{-1})$. The output from the matched filter is then given by

$$\begin{aligned} \boldsymbol{z}(t) &= \boldsymbol{H}^\dagger(q, q^{-1})\boldsymbol{H}(q, q^{-1})\boldsymbol{d}(t) + \boldsymbol{H}^\dagger(q, q^{-1})\boldsymbol{n}(t) \\ &= \left(h_1^*(q)h_1(q^{-1}) + h_2^*(q)h_2(q^{-1})\right)\boldsymbol{d}(t) \\ &\quad + \boldsymbol{v}(t), \end{aligned} \tag{7}$$

where

$$\boldsymbol{v}(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \boldsymbol{H}^\dagger(q, q^{-1})\boldsymbol{n}(t). \tag{8}$$

The components of $\boldsymbol{z}(t) = [z_1(t) z_2(t)]^T$ can be expressed as

$$\begin{aligned} z_1(t) &= \left(h_1^*(q)h_1(q^{-1}) + h_2^*(q)h_2(q^{-1})\right) d_1(t) + v_1(t) \\ z_2(t) &= \left(h_1^*(q)h_1(q^{-1}) + h_2^*(q)h_2(q^{-1})\right) d_2(t) + v_2(t). \end{aligned}$$

The noise sequences, $v_1(t)$ and $v_2(t)$, are uncorrelated. We can see this from the fact that the spectrum of $\boldsymbol{v}(t)$, given by

$$\begin{aligned} \boldsymbol{R_{vv}}(q, q^{-1}) &= \sum_{m=-\infty}^{\infty} E[\boldsymbol{v}(t)\boldsymbol{v}^H(t-m)]q^{-m} \\ &= \boldsymbol{H}^\dagger(q, q^{-1})\boldsymbol{R_{nn}}(q, q^{-1})\boldsymbol{H}(q, q^{-1}) \\ &= \sigma_n^2 \boldsymbol{H}^\dagger(q, q^{-1})\boldsymbol{H}(q, q^{-1}) \\ &= \sigma_n^2 \left(h_1^*(q)h_1(q^{-1}) + h_2^*(q)h_2(q^{-1})\right)\boldsymbol{I} \end{aligned} \tag{9}$$

has no cross terms between $v_1(t)$ and $v_2(t)$. In the third equality in (9) we have used the fact that $\boldsymbol{n}(t)$ is a white vector noise sequence with $\boldsymbol{R_{nn}}(q, q^{-1}) = \sigma_n^2 \boldsymbol{I}$. The problem of detecting the symbol streams $d_1(t)$ and $d_2(t)$ thus decouples.

The channel after matched filter is the same that would be obtained using one transmit antenna and two receive antennas. Therefore this scheme achieves full diversity.[2]

Finally the two signals, $z_1(t)$ and $z_2(t)$, can be fed to an MLSE that will handle the intersymbol interference. Since

[2]However the scheme does not achieve the array gain as in the case of using two receive antennas.

we use an MLSE that utilize the matched filter metric [2], the estimated symbol sequence $\hat{d}_1(t)$ will be the symbol sequence that maximizes the recursively defined matched filter metric.

$$\mu_{MF}(t) = \mu_{MF}(t-1) + \text{Re}\left\{ d_1^*(t)(2z_1(t) - \gamma_0 d_1(t) - 2\sum_{m=1}^{n\gamma} \gamma_m d_1(t-m))\right\}. \quad (10)$$

In (10), γ_k are the coefficients of the double sided complex conjugate symmetric metric polynomial

$$\boldsymbol{\gamma}(q, q^{-1}) = \gamma_{n\gamma}^* q^{n\gamma} + \cdots + \gamma_0 + \cdots + \gamma_{n\gamma} q^{-n\gamma} = h_1^*(q)h_1(q^{-1}) + h_2^*(q)h_2(q^{-1}).$$

The estimated symbol sequence $\hat{d}_2(t)$ is similarly formed by maximizing the corresponding metric utilizing the second component, $z_2(t)$, of $\boldsymbol{z}(t)$.

B Anti-causal signaling

Consider the components $r_1(t)$ and $r_2(t)$ of the vector signal $\boldsymbol{r}(t) = \begin{bmatrix} r_1(t) & r_2(t) \end{bmatrix}^T$:

$$r_1(t) = h_1(q^{-1})d_1(t) + h_2(q^{-1})d_2(t) + n_1(t) \quad (11)$$
$$r_2(t) = h_2^*(q)d_1(t) - h_1^*(q)d_2(t) + n_2(t). \quad (12)$$

To achieve the signal $r_1(t)$ in the first half of the frame we simply transmit the symbol stream $d_1(t)$ from antenna 1 and symbol stream $d_2(t)$ from antenna 2. Achieving $r_2(t)$ at the receiver is less straightforward but nonetheless possible. Consider the two symbol streams $d_1(t)$ and $d_2(t)$. Let us choose their length to be $N+1$. *Time reverse* these symbol streams to form the new symbol streams

$$\tilde{d}_1(t) = d_1(N-t), \quad t = 0, 1, \ldots, N \quad (13)$$
$$\tilde{d}_2(t) = d_2(N-t), \quad t = 0, 1, \ldots, N. \quad (14)$$

Now transmit $-\tilde{d}_2^*(t)$ from antenna 1 and $\tilde{d}_1^*(t)$ from antenna 2. The signal at the receiver will then be

$$r_2'(t) = h_2(q^{-1})\tilde{d}_1^*(t) - h_1(q^{-1})\tilde{d}_2^*(t) + n(t). \quad (15)$$

By time reversing $r_2'(t)$ in (15) and complex conjugating it we obtain the signal

$$(r_2'(N-t))^* = h_2^*(q)d_1(t) - h_1^*(q)d_2(t) + n_2(t). \quad (16)$$

Note that this is the desired signal $r_2(t)$ in (12). We have in (16) denoted $n^*(N-t)$ with $n_2(t)$.

On the receive side, during the first half of the frame, the samples are collected to form the sequence $r_1(t)$ and during the second half of the frame the samples are collected and the sequence is complex conjugated and time reversed in order to form the sequence $r_2(t)$. The sequences $r_1(t)$ and $r_2(t)$ are then fed into the MIMO matched filter $\boldsymbol{H}^\dagger(q, q^{-1})$ to form the decoupled outputs $z_1(t)$ and $z_2(t)$. The sequences $z_1(t)$ and $z_2(t)$ are then used independently to estimate the transmitted sequences $d_1(t)$ and $d_2(t)$. This detection can for example be performed with a maximum likelihood sequence estimator.

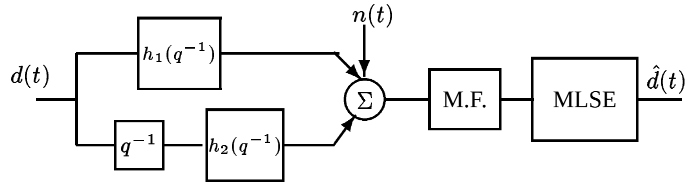

Figure 2: *Block diagram for a delay diversity scheme.*

IV TRANSMIT DELAY DIVERSITY TECHNIQUE

In Figure 2 a transmit delay diversity scheme [4] is depicted. The symbol sequence, $d(t)$ is transmitted by two antennas. From the first antenna the signal is transmitted without a delay and from the second antenna the signal is transmitted with a delay. Here a delay of one symbol interval is used.

At the receiver the signal is filtered by a filter matched to the equivalent channel impulse response and then fed to an MLSE as described for example in Section A.

V TIME REVERSAL SPACE-TIME BLOCK CODING VS. TRANSMIT DELAY DIVERSITY

A simple method that can be used to achieve transmit diversity in a channel with intersymbol interference is the transmit delay diversity method described above. This method however has the drawback that it increases the length of the memory in the channel. The equalization in the receiver can thus become more complex. If an MLSE is used in the receiver then the complexity increase can be significant. The increase in complexity is especially large when signaling with higher order constellations. This will also likely be the case even if a suboptimal receiver is used (as for example a decision feedback sequence estimator). The reason for this is that the delayed transmission will result in extra channel taps with significant mean energy, as opposed to the energy of the normal delayed channel taps. The time reversal space-time block coding scheme presented in [1] however does not increase the length of the memory in the channel. The complexity of the equalization is therefore not significantly increased.

Another advantage with the time reversal space-time block coding scheme is that it achieves the same diversity order as can be achieved by using one transmit and two receive antennas [1]. The scheme thus achieves full diversity.

A drawback with time reversal space-time block coding is that the number of channel parameters that need to be estimated is doubled (i.e., we need to estimate both $h_1(q^{-1})$ and $h_2(q^{-1})$). Moreover, the accuracy in the channel estimation affects the diagonalization of the channel in (7). The detection of the two symbol streams $d_1(t)$ and $d_2(t)$ may thus not completely decouple.

VI SIMULATION RESULTS

The time reversal space-time block coding and the transmit delay diversity methods described above have been applied to a GSM-like system. In particular, we used the same symbol interval as in GSM system, $3.69\mu s$. For the modulation format we used a linearized approximation of the GMSK modulation [5]. For the channel we used the average power delay profiles for the standard GSM scenarios [6]. Figure 3 shows the three typical scenarios we used that were the rural (RTx), the typical urban (TUx) and the hilly terrain (HTx). For each tap we generated a complex gain with an amplitude which was Rayleigh distributed with a given given average power, and an uniformly distributed phase. We assumed the channel to be stationary on a burst period. Moreover the channels were generated independently (i.e. , we assumed no correlation between the two antennas).

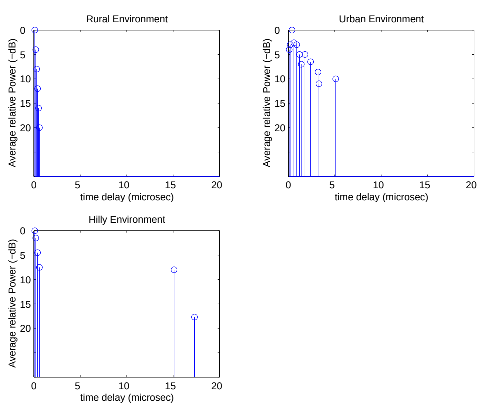

Figure 3: The three typical scenarios.

For the channel estimation we used a reduced parameters channel estimation technique presented in [2]. This method reduces the number of parameters to estimate by using the *a-priori* knowledge on the pulse shape filtering. This is especially useful for the time reversal space-time block coding scheme as it has to estimate twice the number of channel parameters compared to the non transmit diversity case. In both the transmission schemes, we used a training sequence consisting of 28 symbols.

For each operation point we ran 5000 simulations.

The two scheme are compared with a normal transmission scheme with *one transmit antenna* and *one receive antenna*. Figures 4, 5 and 6 shows the BER as a function of input SNR.

For the rural environment, which is where the extra diversity has the largest impact since it is particularly diversity starved to start with (the delay spread is less than $0.5\mu s$), the gain is 5 dB at a BER of 10^{-2} for the TR-STBC scheme and 4 dB for the transmit delay diversity scheme. In the hilly environment which is the hardest one in terms of channel estimation (delay spread until $17.2\mu s$), the performance of the two methods are similar and at 10^{-2} BER the gain is 2 dB. In the urban environment the gain for the TR-STBC scheme is 3 dB and the gain for the transmit delay diversity scheme is 2.5 dB. These gains can deliver valuable coverage and capacity improvements.

The performance results are summarized in the following table:

Gain at 1% BER	Delay Diversity	TR-STBC scheme
Rural	4 dB	5 dB
Urban	2.5 dB	3 dB
Hilly	2 dB	2 dB

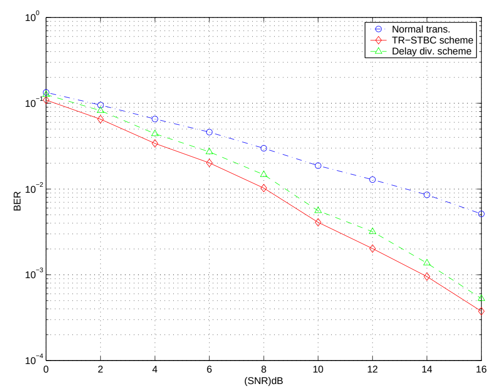

Figure 4: BER for Rural Environment

VII CONCLUSION

In this work we tested the performance of the new transmit diversity algorithm introduced in [1] and we compared it with the performance of a transmit delay diversity technique for some typical scenarios for a GSM-like system. The results show that the new scheme achieves better performance than the transmit delay diversity scheme. For the rural environment, at a BER of 10^{-2}, the TR-STBC scheme has a gain of 5 dB compared to the one transmit antenna and one receive antenna scheme and a gain of 1 dB compared to the transmit

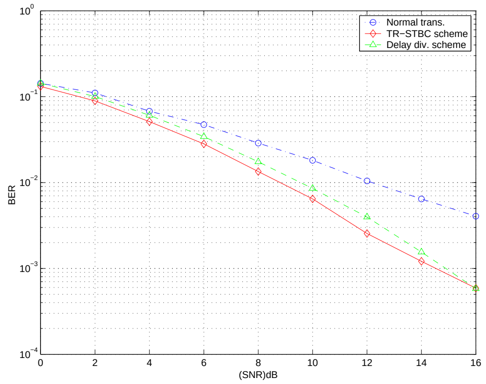

Figure 5: BER for Urban Environment

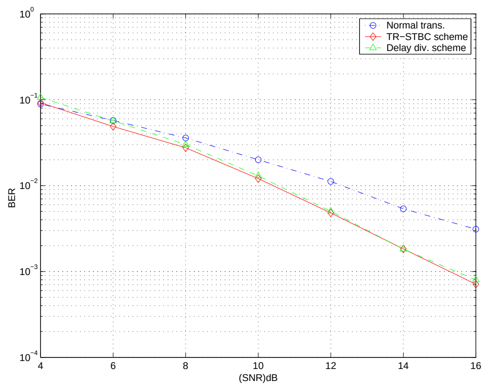

Figure 6: BER for Urban Environment

delay diversity scheme. In the urban case the gains are 3 dB and 0.5 dB respectively. Moreover the TR-STBC scheme can use a simpler equalizer than typically will be required for the transmit delay diversity scheme.

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