PERFORMANCE ANALYSIS OF A SPREAD SPECTRUM CODE ACQUISITION SYSTEM USING A SUBBAND LMS ADAPTIVE FILTER

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ABSTRACT

This paper investigates the performance of the subband LMS algorithm when used to acquire timing in a spread spectrum communication system. This research focuses on the frequency selective channel where multipath interference causes significant input correlation and slow convergence. The subband LMS adaptive filter is used to reduce the effect of the input correlation and to increase the acquisition time. However, experiments show that the subband LMS algorithm actually acquires at a slower rate than the traditional LMS algorithm even though the input correlation is reduced. This paper shows that the reduced performance is due to an increase in the variance of the tap weight estimates. The adaptive filtering approach to timing acquisition uses tap weight values to estimate timing information. The subband LMS algorithm updates tap weights based on blocks (similar to block based LMS). As a result the tap weight variance increases and lower performance results.

1. INTRODUCTION

In a spread spectrum communication system the information bit of each user is encoded with a spreading sequence. In order to retrieve the information of each user, the spreading sequence used for encoding must be known to the receiver. To perform de-spreading of the received signal, both the received signal and spreading sequence (reference signal) must be aligned (code acquisition). There are different techniques used to perform code acquisition such as the sliding correlator (slow, low complexity), parallel bank of correlators (fast, high complexity), and a combination of both (hybrid) [1]. A hybrid code acquisition system such as the Least Mean Square (LMS) adaptive algorithm can provide a trade-off between speed and complexity [2]. Since multipath interference is common in wireless channels where spread spectrum systems are used, the transmitted signal becomes highly correlated. This degrades the performance of the LMS system since it increases the time required to acquire the signal [3]. Lower code acquisition times can be achieved if a reduction in the correlation of the received signal can be obtained. Using subband adaptive filtering the input signal correlation can be reduced. Experimental results show that fast acquisition can be achieved only when the time delay of the received and reference signals is not greater than the length of the adaptive filter. This is hardly the case since the length of the adaptive filter is less than the period of the spreading sequence in a hybrid system. The end result is that the subband LMS (SLMS) system performs more poorly than the LMS system in acquiring the received signal. Further analysis shows that the degradation in performance is due to an increase in variance of the adaptive filter taps. This increase in variance is the result of adapting the filter taps based on a block adaptive algorithm such as the SLMS.

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2. SYSTEM DESCRIPTION

Code acquisition is obtained by estimating the delay between the input and reference code signals. This is achieved by using a FIR digital filter in which the tap weights are updated using an adaptive algorithm. After convergence of the filter, the index of the largest tap is used as the delay estimate. Since for a hybrid acquisition system the number of filter taps used is less than the period of the PN spreading sequence, the actual delay can be located outside the span of the filter. In order to estimate the time delay, the spreading sequence (L chips long) must be divided into cells smaller than the length of the sequence. There are two types of cells depending on the location of the actual delay. If the delay is within the span of the filter the

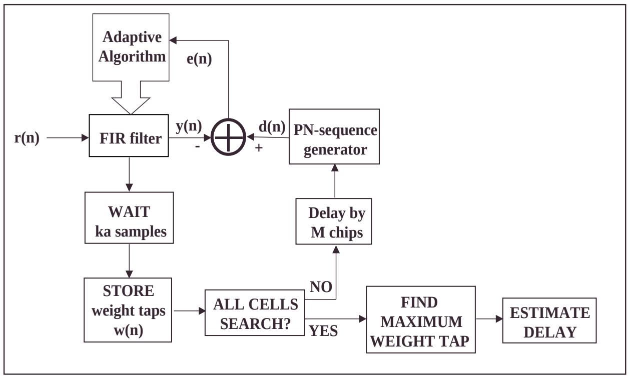

Figure 1: Adaptive code acquisition system.

cell is *in-phase*, and if not then the cell is *out-of-phase*. For a FIR filter of size M there are at most $q = \lceil \frac{L}{M} \rceil$ ($\lceil\rceil$ largest integer) cells. The adaptive code acquisition system searches one cell at a time storing the filter taps in a buffer after ka samples (ka=convergence time of adaptive algorithm) have been processed, and then shifts the reference signal by M chips. This process is repeated q times, re-initializing the filter every time a different cell is processed [2]. The delay is then estimated by locating the index of the largest filter tap in the storage buffer. Figure 1 shows the block diagram of the adaptive acquisition algorithm. As a baseline for comparison, the acquisition performance of the system is computed using the LMS algorithm to update the filter taps. The general form of the LMS algorithm is given by

$$\hat{\mathbf{w}}(n+1) = \hat{\mathbf{w}}(n) + \mu_{lms}\mathbf{x}(n)e^*(n), \quad (1)$$

where $\hat{\mathbf{w}}$ is an estimate of the optimum weight vector given by the Wiener-Hopf solution. The factor μ_{lms} is the step size controlling the convergence speed, and the misadjustment of the algorithm [4]. The convergence time of the adaptive algorithm is defined as the time it takes for the filter taps to converge and reach a steady state condition. The convergence time of the LMS algorithm is given by

$$\tau_{lms} = \frac{2}{\mu_{lms}\lambda_{av}}, \quad (2)$$

where λ_{av} is the average of the eigenvalues of the input autocorrelation matrix [5].

There are different subband filtering structures used for adaptive filtering but most of them require the adaptation of the filter taps in the transform domain. To estimate the time delay of the input signal, the filter taps must be updated in the time domain. Such a structure is provided by the subband LMS algorithm

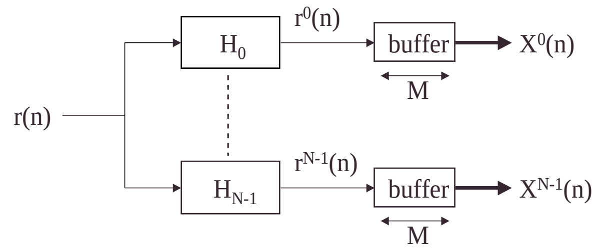

Figure 2: Subband filtering of input signal.

proposed by [6]. This adaptive algorithm divides the input and output error signals into N subbands using a filterbank. Each subband is then adapted using the LMS criteria. The filterbank used is based on the Modulated Lapped Transform [7], which is given by the following equations:

$$\mathbf{h_i}(n) = \mathbf{p}(n)\sqrt{\frac{2}{N}}\cos\left[(i+\frac{1}{2})(n+\frac{N+1}{2})\frac{\pi}{N}\right], \quad (3)$$

where $i = 0, 1, \ldots, N-1$, and $n = 0, 1, \ldots, 2N-1$ and the baseband filter $p(n)$ is given by

$$\mathbf{p}(n) = -\sin\left[(n+\frac{1}{2})\frac{\pi}{2N}\right].$$

The weight update algorithm for the SLMS is given by the following equation:

$$\mathbf{W^*}_{(k+1)N} = \mathbf{W^*}_{kN} + \mu_{slms}\sum_{i=0}^{N-1}\alpha_i \mathbf{X}^{i^*}_{kN} e^i_k, \quad (4)$$

where $\mathbf{X}^i_{kN}$ is the output of the ith subfilter applied to the input signal (Fig. 2), and e^i_k is the ith decimated subband component of the filter output error (Fig. 3). A reduction in the correlation of the input signal is achieved by normalizing each subband by the total power found in that band using:

$$\alpha_i = \frac{1}{M\sigma^2_{x_i}}, \quad (5)$$

where $\sigma^2_{x_i}$ is the power in the ith subband.

Since the SLMS algorithm uses a decimator at the output of the filterbank, the filter taps are updated every N samples. Thus, the SLMS is a block algorithm where the size of the block is equal to the number of subbands. The convergence time of the SLMS algorithm is then given by

$$\tau_{slms} = \frac{2N}{\mu_{slms}\lambda^s_{av}}. \quad (6)$$

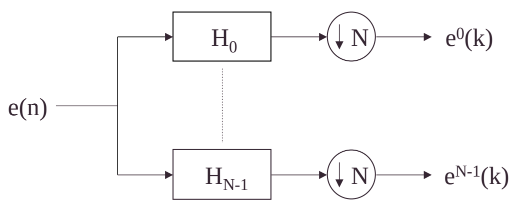

Figure 3: Subband filtering of error signal.

The factor λ_{av}^{s} is the average eigenvalue of the matrix $\mathbf{R}_{slms}$. This is the autocorrelation matrix of the input signal after subband filtering which is given by

$$\mathbf{R}_{slms} = \sum_{i=0}^{N-1} \alpha_i \mathbf{R}_{X^i X^i}, \qquad (7)$$

where $\mathbf{R}_{X^i X^i}$ is the auto-correlation matrix of the output of each subband [6].

3. EXPERIMENT SETUP

A PN spreading sequence with period 127 chips is used, and passed through a 3-ray Multipath Rayleigh Fading channel. Additive White Gaussian Noise is added at the output of the channel. The signal and channel power are normalized to unity, so that the signal to noise ratio of the signal is $\frac{1}{\sigma_n^2}$ where σ_n^2 is the variance of the AWGN. The received signal is sampled at the chip rate, and then the output of the sampler is used as input to the acquisition system based on the LMS and SLMS algorithms. A FIR filter of 16 taps is used throughout all the simulations, resulting in $q = 8$ cells to be processed. Two and four subbands are used in the SLMS system simulation. To measure the performance of the system 1000 experiments are conducted for each set of input parameters: SNR, step-size, and a random time delay that is uniformly distributed among the code period (Fig. 4). For all experiments, statis-

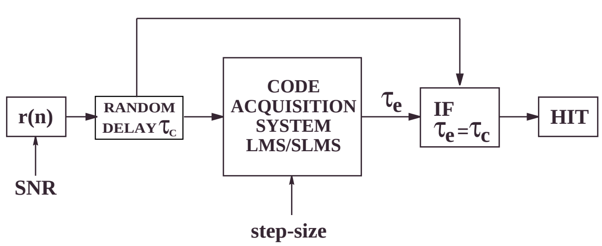

Figure 4: System setup.

tics are collected on the output parameters: estimated delay and time of acquisition. The average time of acquisition is obtained by averaging all experiments using a probability of false acquisition $\mathbf{P}_{FA} \leq 0.05$.

4. SIMULATION RESULTS

The introduction of a Multipath Rayleigh Fading channel correlates the PN spreading sequence degrading the time acquisition performance of the adaptive system. Figure 5 shows the performance of the LMS code acquisition system with and without multipath interference. The LMS is clearly slower when the multipath channel

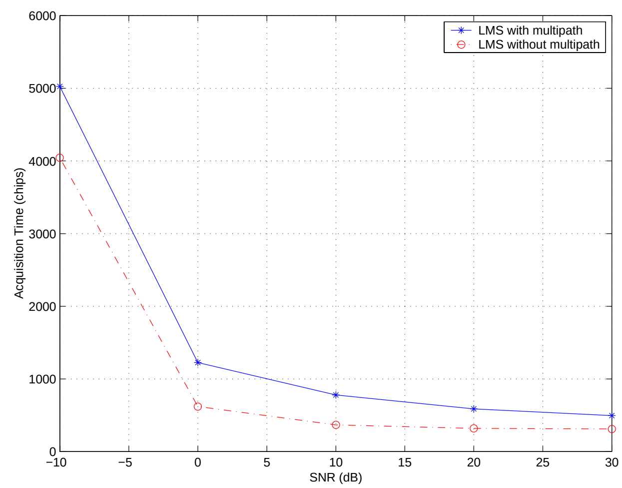

Figure 5: Performance of LMS system.

is present. A measure of the correlation introduced by the multipath channel can be given by the eigenvalue spread $(\frac{\lambda_{max}}{\lambda_{min}})$ of the autocorrelation matrix controlling the adaptive algorithm. The spread of the input signal with multipath channel is shown in Figure 6. Also shown, is the spread of the input signal after subband processing with two and four subbands. The results show a reduction in spread with the use of the SLMS algorithm, and higher reductions are obtained when the number of subbands is increased. However, the reduction only occurs at high SNR's since noise dominates at low SNR's reducing the correlation in the signal. The performance of the SLMS acquisition system compared to the LMS system is shown in Figure 7. As the number of subbands is increased the performance of the SLMS algorithm degrades, requiring a longer time to acquire the spreading sequence. In addition, the SLMS acquisition performance is slower than the LMS system. To explain the slower performance shown by the SLMS algorithms, the *square deviation error* (tap weight error) of the filter taps is computed for each adaptive algorithm by setting the convergence time constant of each algorithm equal to each other ($\tau_{lms} = \tau_{slms}$). The tap

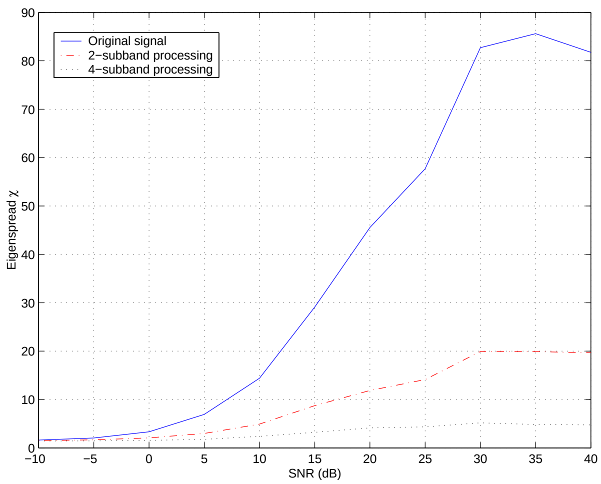

Figure 6: Eigenvalue spread of input signal.

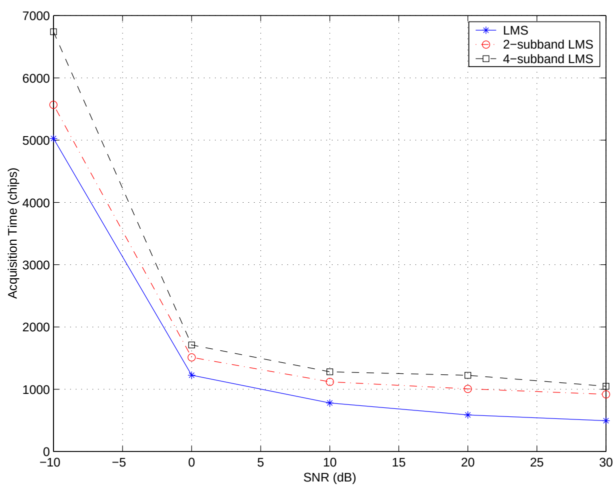

Figure 7: Performance of LMS and SLMS acquisition systems.

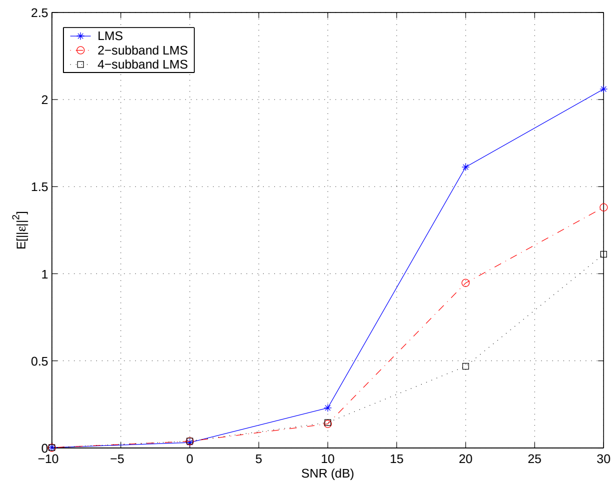

Figure 8: In-phase cell tap weight error.

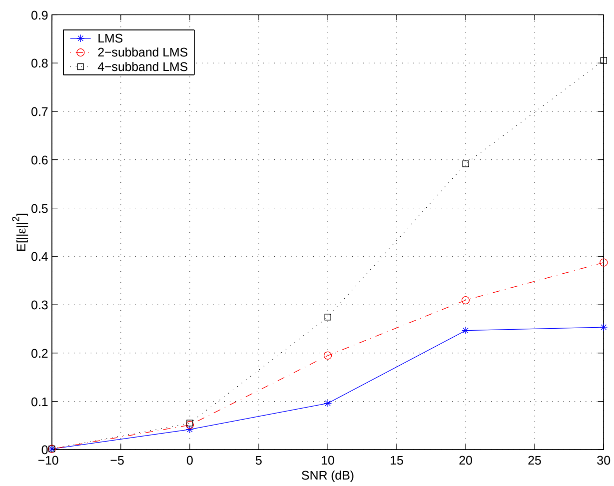

Figure 9: Out-of-phase cell tap weight error.

weight error results of the *in-phase* cell show a behavior that agrees with the reduction of the input signal eigenvalue spread. As the number of subbands increases, the error is lower compared to the LMS algorithm (Fig. 8). However, the results of the *ouf-of-phase* cell show that the error increases as the number of subbands is increased. The weight error is also higher than the LMS weight error (Fig. 9). Block algorithms are known for having a faster acquisition time but at the expense of higher variance in the filter taps [8]. The higher variance introduced by the block process dominates the spread reduction gain in the *out-of-phase* cell. Since there are more of these cells, the overall effect results in a slower acquisition performance.

5. CONCLUSIONS

A new spread spectrum code acquisition system based on the SLMS algorithm was implemented. The average acquisition time performance of the filter was obtained using a filterbank of two and four subbands, and compared to the LMS acquisition system. The eigenvalue spread of the received signal, transmitted over a Multipath Rayleigh Fading channel of 3-paths, indicated a highly correlated signal for high signal to noise ratios, and only a small spread for low SNR's. As initially predicted, the use of a subband adaptive filter applied to the received signal dramatically reduced the eigenspread of the original process. It was believed originally that the reduction in spread would result in faster convergence of the SLMS adaptive filter, and consequently a faster average acquisition time. However, after comparing the performance plots for the LMS and SLMS systems, the SLMS required a higher average acquisition time to achieve the same probability of acquisition. Further insight into the weigth tap error (squared error deviation) metric reveals an interesting behavior. When the delay between the received and reference signals is within the span of the filter (*in-phase* cell), the SLMS filter behaves as predicted. The higher the reduction in eigenspread due to an increase in the number of subbands, the smaller the weigth tap error compared to the LMS case. In other words, the SLMS *in-phase* cell can converge faster than the LMS *in-phase* cell with the same weigth tap error. However, when the delay between the received and reference signals is not within the span of the filter (*out-of-phase* cell), the tap weight error of the SLMS filter is larger than in the LMS case. In addition, as the number of subbands increases, so does the tap weight error. This behavior can be explained by recalling that the SLMS algorithm is processed in blocks, rather than samples like the LMS. The problem with a block algorithm is that the deviation of the weigth taps around the optimum Wiener-Hopf solution increases with increasing block size. This block error is present in both the *in-phase* and the *out-of-phase* cells, but dominates in the *out-of-phase* cell. Since there are more *out-of phase* cells, the probability of estimating the wrong delay is highly increased. By lowering the value of the μ_{slms}, the tap weight error is reduced in both cells, effectively increasing the probability of acquisition but at the cost of a higher acquisition time.

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